HANDBOOK of APPLIED CRYPTOGRAPHY

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To Archie and Lida Menezes

To Cornelis Henricus van Oorschot
and Maria Anna Buys van Vugt

To Margaret and Gordon Vanstone

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Foreword

by R.L. Rivest

As we draw near to closing out the twentieth century, we see quite clearly that the information-processing and telecommunications revolutions now underway will continue vigorously into the twenty-first. We interact and transact by directing flocks of digital packets towards each other through cyberspace, carrying love notes, digital cash, and secret corporate documents. Our personal and economic lives rely more and more on our ability to let such ethereal carrier pigeons mediate at a distance what we used to do with face-to-face meetings, paper documents, and a firm handshake. Unfortunately, the technical wizardry enabling remote collaborations is founded on broadcasting everything as sequences of zeros and ones that one's own dog wouldn't recognize. What is to distinguish a digital dollar when it is as easily reproducible as the spoken word? How do we converse privately when every syllable is bounced off a satellite and smeared over an entire continent? How should a bank know that it really *is* Bill Gates requesting from his laptop in Fiji a transfer of $10,000,000,000 to another bank? Fortunately, the magical mathematics of cryptography can help. Cryptography provides techniques for keeping information secret, for determining that information has not been tampered with, and for determining who authored pieces of information.

Cryptography is fascinating because of the close ties it forges between theory and practice, and because today's practical applications of cryptography are pervasive and critical components of our information-based society. Information-protection protocols designed on theoretical foundations one year appear in products and standards documents the next. Conversely, new theoretical developments sometimes mean that last year's proposal has a previously unsuspected weakness. While the theory is advancing vigorously, there are as yet few true guarantees; the security of many proposals depends on unproven (if plausible) assumptions. The theoretical work refines and improves the practice, while the practice challenges and inspires the theoretical work. When a system is "broken," our knowledge improves, and next year's system is improved to repair the defect. (One is reminded of the long and intriguing battle between the designers of bank vaults and their opponents.)

Cryptography is also fascinating because of its game-like adversarial nature. A good cryptographer rapidly changes sides back and forth in his or her thinking, from attacker to defender and back. Just as in a game of chess, sequences of moves and counter-moves must be considered until the current situation is understood. Unlike chess players, cryptographers must also consider all the ways an adversary might try to gain by breaking the rules or violating expectations. (Does it matter if she measures how long I am computing? Does it matter if her "random" number isn't one?)

The current volume is a major contribution to the field of cryptography. It is a rigorous encyclopedia of known techniques, with an emphasis on those that are both (believed to be) secure and practically useful. It presents in a coherent manner most of the important cryptographic tools one needs to implement secure cryptographic systems, and explains many of the cryptographic principles and protocols of existing systems. The topics covered range from low-level considerations such as random-number generation and efficient modular exponentiation algorithms and medium-level items such as public-key signature techniques, to higher-level topics such as zero-knowledge protocols. This book's excellent organization and style allow it to serve well as both a self-contained tutorial and an indispensable desk reference.

In documenting the state of a fast-moving field, the authors have done incredibly well at providing error-free comprehensive content that is up-to-date. Indeed, many of the chapters, such as those on hash functions or key-establishment protocols, break new ground in both their content and their unified presentations. In the trade-off between comprehensive coverage and exhaustive treatment of individual items, the authors have chosen to write simply and directly, and thus efficiently, allowing each element to be explained together with their important details, caveats, and comparisons.

While motivated by practical applications, the authors have clearly written a book that will be of as much interest to researchers and students as it is to practitioners, by including ample discussion of the underlying mathematics and associated theoretical considerations. The essential mathematical techniques and requisite notions are presented crisply and clearly, with illustrative examples. The insightful historical notes and extensive bibliography make this book a superb stepping-stone to the literature. (I was very pleasantly surprised to find an appendix with complete programs for the CRYPTO and EUROCRYPT conferences!)

It is a pleasure to have been asked to provide the foreword for this book. I am happy to congratulate the authors on their accomplishment, and to inform the reader that he/she is looking at a landmark in the development of the field.

Ronald L. Rivest
Webster Professor of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
August 1996

Preface

This book is intended as a reference for professional cryptographers, presenting the techniques and algorithms of greatest interest to the current practitioner, along with the supporting motivation and background material. It also provides a comprehensive source from which to learn cryptography, serving both students and instructors. In addition, the rigorous treatment, breadth, and extensive bibliographic material should make it an important reference for research professionals.

Our goal was to assimilate the existing cryptographic knowledge of industrial interest into one consistent, self-contained volume accessible to engineers in practice, to computer scientists and mathematicians in academia, and to motivated non-specialists with a strong desire to learn cryptography. Such a task is beyond the scope of each of the following: research papers, which by nature focus on narrow topics using very specialized (and often non-standard) terminology; survey papers, which typically address, at most, a small number of major topics at a high level; and (regretably also) most books, due to the fact that many book authors lack either practical experience or familiarity with the research literature or both. Our intent was to provide a detailed presentation of those areas of cryptography which we have found to be of greatest practical utility in our own industrial experience, while maintaining a sufficiently formal approach to be suitable both as a trustworthy reference for those whose primary interest is further research, and to provide a solid foundation for students and others first learning the subject.

Throughout each chapter, we emphasize the relationship between various aspects of cryptography. Background sections commence most chapters, providing a framework and perspective for the techniques which follow. Computer source code (e.g. C code) for algorithms has been intentionally omitted, in favor of algorithms specified in sufficient detail to allow direct implementation without consulting secondary references. We believe this style of presentation allows a better understanding of how algorithms actually work, while at the same time avoiding low-level implementation-specific constructs (which some readers will invariably be unfamiliar with) of various currently-popular programming languages.

The presentation also strongly delineates what has been established as fact (by mathematical arguments) from what is simply current conjecture. To avoid obscuring the very applied nature of the subject, rigorous proofs of correctness are in most cases omitted; however, references given in the Notes section at the end of each chapter indicate the original or recommended sources for these results. The trailing Notes sections also provide information (quite detailed in places) on various additional techniques not addressed in the main text, and provide a survey of research activities and theoretical results; references again indicate where readers may pursue particular aspects in greater depth. Needless to say, many results, and indeed some entire research areas, have been given far less attention than they warrant, or have been omitted entirely due to lack of space; we apologize in advance for such major omissions, and hope that the most significant of these are brought to our attention.

To provide an integrated treatment of cryptography spanning foundational motivation through concrete implementation, it is useful to consider a hierarchy of thought ranging from conceptual ideas and end-user services, down to the tools necessary to complete actual implementations. Table 1 depicts the hierarchical structure around which this book is organized. Corresponding to this, Figure 1 illustrates how these hierarchical levels map

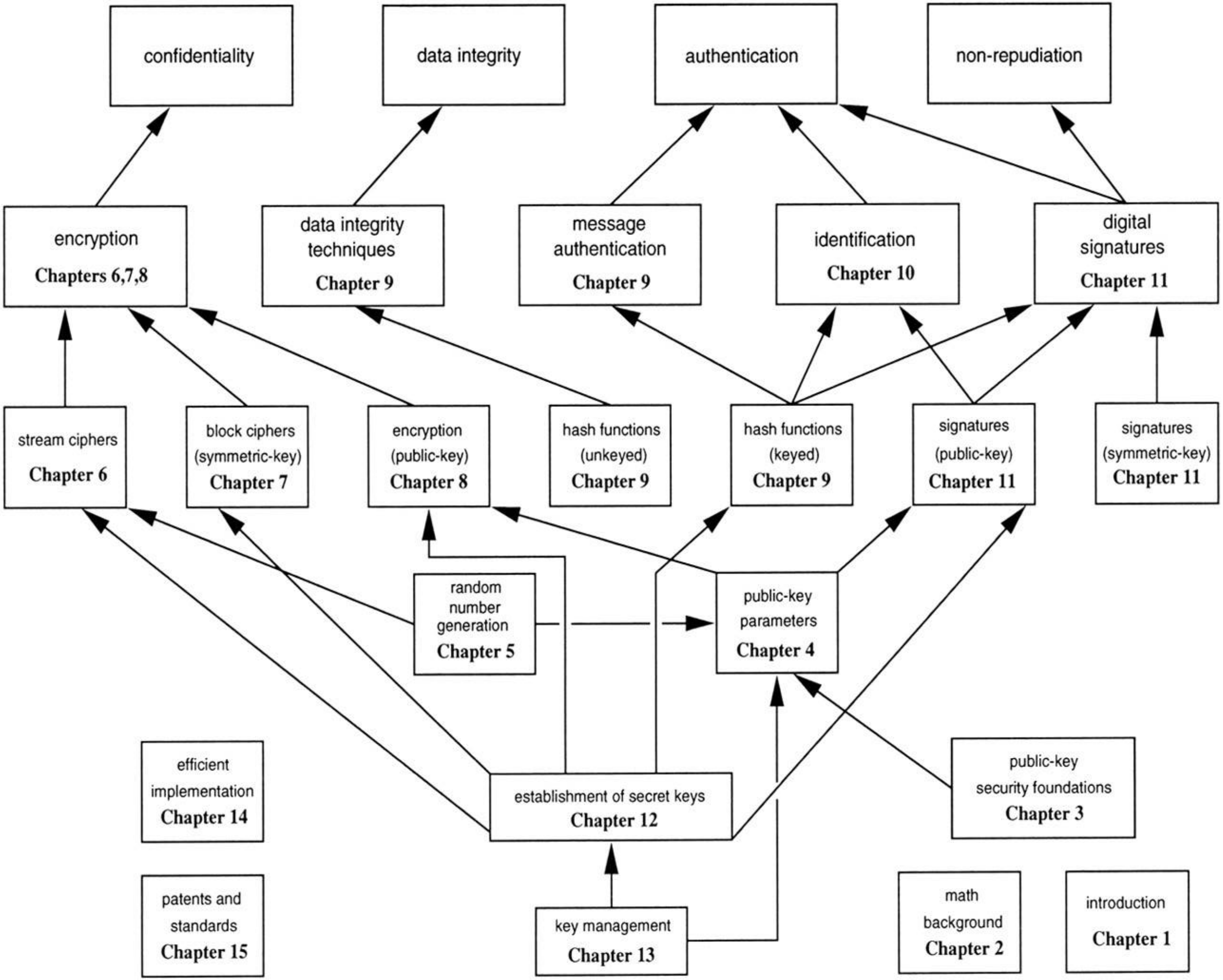
Information Security Objectives	
Confidentiality	
Data integrity	
Authentication (entity and data origin)	
Non-repudiation	
Cryptographic functions	
Encryption	Chapters 6, 7, 8
Message authentication and data integrity techniques	Chapter 9
Identification/entity authentication techniques	Chapter 10
Digital signatures	Chapter 11
Cryptographic building blocks	
Stream ciphers	Chapter 6
Block ciphers (symmetric-key)	Chapter 7
Public-key encryption	Chapter 8
One-way hash functions (unkeyed)	Chapter 9
Message authentication codes	Chapter 9
Signature schemes (public-key, symmetric-key)	Chapter 11
Utilities	
Public-key parameter generation	Chapter 4
Pseudorandom bit generation	Chapter 5
Efficient algorithms for discrete arithmetic	Chapter 14
Foundations	
Introduction to cryptography	Chapter 1
Mathematical background	Chapter 2
Complexity and analysis of underlying problems	Chapter 3
Infrastructure techniques and commercial aspects	
Key establishment protocols	Chapter 12
Key installation and key management	Chapter 13
Cryptographic patents	Chapter 15
Cryptographic standards	Chapter 15

Table 1: *Hierarchical levels of applied cryptography.*

onto the various chapters, and their inter-dependence.

Table 2 lists the chapters of the book, along with the primary author(s) of each who should be contacted by readers with comments on specific chapters. Each chapter was written to provide a self-contained treatment of one major topic. Collectively, however, the chapters have been designed and carefully integrated to be entirely complementary with respect to definitions, terminology, and notation. Furthermore, there is essentially no duplication of material across chapters; instead, appropriate cross-chapter references are provided where relevant.

While it is not intended that this book be read linearly from front to back, the material has been arranged so that doing so has some merit. Two primary goals motivated by the "handbook" nature of this project were to allow easy access to stand-alone results, and to allow results and algorithms to be easily referenced (e.g., for discussion or subsequent cross-reference). To facilitate the ease of accessing and referencing results, items have been categorized and numbered to a large extent, with the following classes of items jointly numbered consecutively in each chapter: *Definitions*, *Examples*, *Facts*, *Notes*, *Remarks*, *Algorithms*, *Protocols*, and *Mechanisms*. In more traditional treatments, *Facts* are usually identified as propositions, lemmas, or theorems. We use numbered *Notes* for additional technical points,


confidentiality
data integrity
authentication
non-repudiation
encryption
Chapters 6,7,8
data integrity techniques
Chapter 9
message authentication
Chapter 9
identification
Chapter 10
digital signatures
Chapter 11
stream ciphers
Chapter 6
block ciphers (symmetric-key)
Chapter 7
encryption (public-key)
Chapter 8
hash functions (unkeyed)
Chapter 9
hash functions (keyed)
Chapter 9
signatures (public-key)
Chapter 11
signatures (symmetric-key)
Chapter 11
random number generation
Chapter 5
public-key parameters
Chapter 4
efficient implementation
Chapter 14
establishment of secret keys
Chapter 12
public-key security foundations
Chapter 3
patents and standards
Chapter 15
key management
Chapter 13
math background
Chapter 2
introduction
Chapter 1

	Chapter	Primary Author		
		AJM	PVO	SAV
1.	Overview of Cryptography	*	*	*
2.	Mathematical Background	*		
3.	Number-Theoretic Reference Problems	*		
4.	Public-Key Parameters	*	*	
5.	Pseudorandom Bits and Sequences	*		
6.	Stream Ciphers	*		
7.	Block Ciphers		*	
8.	Public-Key Encryption	*		
9.	Hash Functions and Data Integrity		*	
10.	Identification and Entity Authentication		*	
11.	Digital Signatures			*
12.	Key Establishment Protocols		*	
13.	Key Management Techniques		*	
14.	Efficient Implementation			*
15.	Patents and Standards		*	
—	Overall organization	*	*	

Table 2: *Primary authors of each chapter.*

while numbered *Remarks* identify non-technical (often non-rigorous) comments, observations, and opinions. *Algorithms*, *Protocols* and *Mechanisms* refer to techniques involving a series of steps. *Examples*, *Notes*, and *Remarks* generally begin with parenthetical summary titles to allow faster access, by indicating the nature of the content so that the entire item itself need not be read in order to determine this. The use of a large number of small subsections is also intended to enhance the handbook nature and accessibility to results.

Regarding the partitioning of subject areas into chapters, we have used what we call a *functional organization* (based on functions of interest to end-users). For example, all items related to entity authentication are addressed in one chapter. An alternative would have been what may be called an *academic organization*, under which perhaps, all protocols based on zero-knowledge concepts (including both a subset of entity authentication protocols and signature schemes) might be covered in one chapter. We believe that a functional organization is more convenient to the practitioner, who is more likely to be interested in options available for an entity authentication protocol (Chapter 10) or a signature scheme (Chapter 11), than to be seeking a zero-knowledge protocol with unspecified end-purpose.

In the front matter, a top-level Table of Contents (giving chapter numbers and titles only) is provided, as well as a detailed Table of Contents (down to the level of subsections, e.g., §5.1.1). This is followed by a List of Figures, and a List of Tables. At the start of each chapter, a brief Table of Contents (specifying section number and titles only, e.g., §5.1, §5.2) is also given for convenience.

At the end of the book, we have included a list of papers presented at each of the Crypto, Eurocrypt, Asiacrypt/Auscrypt and Fast Software Encryption conferences to date, as well as a list of all papers published in the *Journal of Cryptology* up to Volume 9. These are in addition to the *References* section, each entry of which is cited at least once in the body of the handbook. Almost all of these references have been verified for correctness in their exact titles, volume and page numbers, etc. Finally, an extensive Index prepared by the authors is included. The Index begins with a List of Symbols.

Our intention was not to introduce a collection of new techniques and protocols, but rather to selectively present techniques from those currently available in the public domain. Such a consolidation of the literature is necessary from time to time. The fact that many good books in this field include essentially no more than what is covered here in Chapters 7, 8 and 11 (indeed, these might serve as an introductory course along with Chapter 1) illustrates that the field has grown tremendously in the past 15 years. The mathematical foundation presented in Chapters 2 and 3 is hard to find in one volume, and missing from most cryptography texts. The material in Chapter 4 on generation of public-key parameters, and in Chapter 14 on efficient implementations, while well-known to a small body of specialists and available in the scattered literature, has previously not been available in general texts. The material in Chapters 5 and 6 on pseudorandom number generation and stream ciphers is also often absent (many texts focus entirely on block ciphers), or approached only from a theoretical viewpoint. Hash functions (Chapter 9) and identification protocols (Chapter 10) have only recently been studied in depth as specialized topics on their own, and along with Chapter 12 on key establishment protocols, it is hard to find consolidated treatments of these now-mainstream topics. Key management techniques as presented in Chapter 13 have traditionally not been given much attention by cryptographers, but are of great importance in practice. A focused treatment of cryptographic patents and a concise summary of cryptographic standards, as presented in Chapter 15, are also long overdue.

In most cases (with some historical exceptions), where algorithms are known to be insecure, we have chosen to leave out specification of their details, because most such techniques are of little practical interest. Essentially all of the algorithms included have been verified for correctness by independent implementation, confirming the test vectors specified.

Acknowledgements

This project would not have been possible without the tremendous efforts put forth by our peers who have taken the time to read endless drafts and provide us with technical corrections, constructive feedback, and countless suggestions. In particular, the advice of our Advisory Editors has been invaluable, and it is impossible to attribute individual credit for their many suggestions throughout this book. Among our Advisory Editors, we would particularly like to thank:

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Any errors that remain are, of course, entirely our own. We would be grateful if readers who spot errors, missing references or credits, or incorrectly attributed results would contact us with details. It is our hope that this volume facilitates further advancement of the field, and that we have helped play a small part in this.

Alfred J. Menezes
Paul C. van Oorschot
Scott A. Vanstone

Preface to the 5th printing

The 5th printing includes corrections to all the editorial and technical errors that we are aware of as of June 2001. We thank everyone for the tremendous reception they have given to our book, and for those who have taken the time to draw errors to our attention.

Alfred J. Menezes
Paul C. van Oorschot
Scott A. Vanstone
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